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RADIATIVE—CONVECTIVE HEAT TRANSFER IN A SYSTEM OF TWO POROUS PLATES

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The solution to a system of equations is investigated, which describe the process of heat transfer between a heated gas and porous plates and which consider the convective heat transfer and the radiation of the particles in the porous plates.

A method is proposed to obtain superadiabatic temperatures in the combustion zone during the burning of a low-calorie gaseous fuel in a system of two highly porous plates, due to the recuperation of thermal energy by the skeletons of the porous plates.

The idea of recovering the heat by radiation in the working zone was suggested by Japanese scientists R. Echigo, Y. Yoshizava, et al. [1, 2]. The one-dimensional transient problem of burning gaseous fuel in porous material was solved [1] considering convective and radiative heat transfer. The combustion rate was found from the Arrhenius equation. The initial temperature profile of the gas was used for the initial condition. The effect was studied of the optical thickness, the absorption coefficient, and the position of the reaction zone relative to the porous layer boundaries on the maximum gas temperature. An original furnace design was proposed [2] for realizing a catalytic reaction, the energy for which was fed in by radiation through a screen transparent to radiation from the particles of the porous layer. The air was heated internally by combustion of an air mixture with a low-grade fuel. The screen is not permissible to material.

Here we examine the method of recuperating the heat by radiation within a system of two highly porous plates. The low-calorie gaseous fuel (diluted natural gas, paint vapors, etc.) is fed through one plate into a narrow gap, where it is burned. The combustion products filter through the second plate. It is proposed to utilize the method of radiative heat transfer to heat the porous plate, through which air is pumped to preheat it by heat transfer between the gas and the skeleton of the porous plate. Preheating the incoming gas permits temperatures above the adiabatic temperature. Only the thermal balance problem has been examined [2], without specifying the chemical and kinetic properties of the system.

A stationary process is examined. The flow of gas is assumed constant. The characteristic dimensions of the plates are much larger than the distance between them, which makes it possible to neglect edge effects and examine a one-dimensional problem. The optical characteristics are taken to be those of a system of spheres of identical radius R_i and emissivity ε_i [3]. The gas flowing through the plates is assumed to be optically transparent. Thermal conduction processes are neglected through the gas and between the particles of the porous plates. Heat transfer occurs by radiation and heat transfer with a heat trans-

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TABLE 1. Variation of DT and κ_i for the Following Parameter Values: G = 0.1 kg/(m²·sec), ε_i = 0.7, Π_i = 0.8, T_0 = 300 K, and L_i = 0.01 m

$\varkappa_i$, W/(m³·K)	10	10^2	10^3	10^4	$5\cdot10^4$	10^5	$5\cdot10^5$	10^6
				$DT/T_0=1$				
T_{max}/T_0	2,000	2,001	2,013	2,058	2,094	2,106	2,120	2,122
T_{out}/T_0	1,999	1,994	1,974	1,929	1,899	1,890	1,880	1,879
				$DT/T_0=2$				
T_{max}/T_0	3,000	3,001	3,029	3,185	3,407	3,512	3,697	3,733
T_{out}/T_0	2,998	2,987	2,935	2,775	2,635	2,591	2,533	2,522
				$DT/T_0=3$				
T_{max}/T_0	4,000	4,002	4,044	4,354	4,937	5,269	5,915	6,074
T_{out}/T_0	3,998	3,981	3,889	3,555	3,222	3,123	2,993	2,968
				$DT/T_0=4$				
T_{max}/T_0	5,000	5,002	5,058	5,538	6,562	7,137	8,068	8,239
T_{out}/T_0	4,998	4,973	4,837	4,288	3,698	3,540	3,338	3,299
				$DT/T_0=5$				
T_{max}/T_0	6,000	6,003	6,069	6,722	8,182	8,925	9,868	10,006
T_{out}/T_0	5,996	5,897	5,781	4,984	4,094	3,879	3,615	3,564
				$DT/T_0=7$				
T_{max}/T_0	8,000	8,006	8,089	9,059	11,24	12,12	12,94	13,049
T_{out}/T_0	7,992	7,890	7,661	6,290	4,723	4,403	4,038	3,967

fer coefficient κ_i between the gas and the particles of the porous layers. Then the energy equation for the gas in the porous plates takes the following form

$$\frac{dT_i}{dx_i} = \frac{\varkappa_i(\vartheta_i - T_i)}{C_p G} \quad (i = 1,\ 2) \tag{1}$$

with initial conditions

$$T_1(x_1 = 0) = T_0,\ T_2(x_2 = 0) = T_1(x_1 = L_1) + DT. \tag{2}$$

Radiation, which leaves a unit volume of the porous plates is a combination of the intrinsic radiation of a unit volume F^c and the external radiation F^v, which is reflected by the volume.

The approach of G. E. Gorelik et al. [3] is used to describe the radiation propagation in the porous bodies. The radiation flux which falls on the separation volume is written as

$$F_i^v(x_i/\lambda_i) = \frac{2\Pi_i J_{0i} E_2(x_i/\lambda_i)}{\lambda_i} + \frac{2\Pi_i J_{1i} E_2\{(L_i - x_i)/\lambda_i\}}{\lambda_i} + 0{,}5\int\limits_0^{L_i} F_i(\xi)\, E_1\{|x_i - \xi|/\lambda_i\}\, d\xi/\lambda_i, \tag{3}$$

where J_{0i} is the radiation flux which falls on the plates from the left; J_{1i} is the radiation flux which falls on the plates from the right; and $\lambda_i = \dfrac{4\Pi_i R_i}{3(1-\Pi_i)}$ is the mean free path of photons.

The expression for the intrinsic radiation of a unit volume of porous plate is

$$F_i^c(x_i/\lambda_i) = S_i \varepsilon_i \sigma \vartheta_i^4, \tag{4}$$

where S_i is the radiating surface of a unit volume of porous plate. The following expression is valid for S_i:

$$S_i = 3(1 - \Pi_i)/R_i, \tag{5}$$

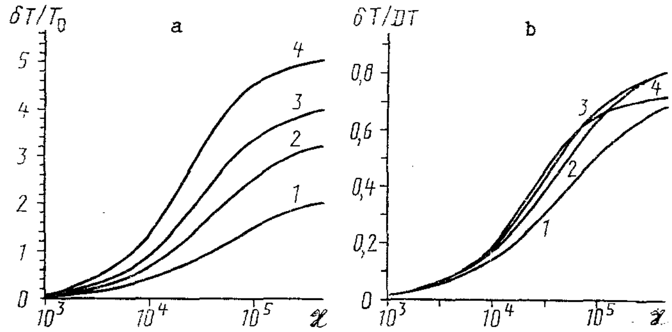

Fig. 1. Increase of the gas temperature δT (K), normalized to T_0 (a) or DT (b), as a function of the heat transfer coefficient κ [W/(kg·m³)]: 1) $DT/T_0 = 3$; 2) $DT/T_0 = 4$; 3) $DT/T_0 = 5$; 4) $DT/T_0 = 7$.

where $(1 - \Pi_i)$ is the volume occupied by particles in a unit volume of the porous layer.

By using the expression for λ_i, S_i can be expresed differently

$$S_i = 4\Pi_i/\lambda_i. \tag{6}$$

The integral equation for the energy of radiation which leaves a unit volume of porous plate is

$$F_i(x_i/\lambda_i) = (1-\varepsilon_i)F_i^v(x_i/\lambda_i) + F_i^c(x_i/\lambda_i). \tag{7}$$

Substitution of (3) yields

$$F_i(x_i/\lambda_i) = \frac{(1-\varepsilon_i)2\Pi_i J_{0i}E_2(x_i/\lambda_i)}{\lambda_i} + \frac{(1-\varepsilon_i)2\Pi_i J_{1i}E_2\{(L_i-x_i)/\lambda_i\}}{\lambda_i} + $$
$$+ 0{,}5(1-\varepsilon_i)\int_0^{L_i} F_i(\xi)E_1\{|x_i-\xi|/\lambda_i\}\,d\xi/\lambda_i + 4\Pi_i\varepsilon_i\sigma\vartheta_i^4/\lambda_i. \tag{8}$$

The dividing volume obtains energy by absorption of external radiation. The energy given up by the volume is spent on the internal radiation of the volume and heat transfer between the volume and the gas flowing through it. The energy balance is written in the form

$$\frac{\varepsilon_i 2\Pi_i J_{0i}E_2(x_i/\lambda_i)}{\lambda_i} + \frac{\varepsilon_i 2\Pi_i J_{1i}E_2\{(L_i-x_i)/\lambda_i\}}{\lambda_i} + $$
$$+ 0{,}5\varepsilon_i\int_0^{L_i} F_i(\xi)E_1\{|x_i-\xi|/\lambda_i\}\,d\xi/\lambda_i = \frac{4\Pi_i\varepsilon_i\sigma\vartheta_i^4}{\lambda_i} + \varkappa_i(\vartheta_i - T_i). \tag{9}$$

The radiation flux leaving from the second plate at $x_2 = 0$ is written as

$$J_{11} = 0{,}5\int_{L_2}^{0} F_2(\xi)E_2(\xi/\lambda_2)\,d\xi/\lambda_2 + 2\Pi_2 J_{12}E_3(L_2/\lambda_2) + (1-\Pi_2)(1-\varepsilon_2)J_{02} + (1-\Pi_2)(1-\varepsilon_2)\sigma\vartheta_2^4(0) \tag{10}$$

and from the first plate at $x_1 = L_1$ as

$$J_{02} = 0{,}5\int_0^{L_1} F_1(\xi)E_2(\xi/\lambda_1)\,d\xi/\lambda_1 + 2\Pi_1 J_{01}E_3(L_1/\lambda_1) + (1-\Pi_1)(1-\varepsilon_1)J_{11} + (1-\Pi_1)(1-\varepsilon_1)\sigma\vartheta_1^4(L_1). \tag{11}$$

The problem is reduced to solving a system of four integral equations and two differential equations. A finite-difference analog of the problem gives a system of nonlinear algebraic equations which are linearized by Newton's rule.

Numerical modeling was conducted by varying various problem parameters. The variation was investigated in the maximum gas temperature T_i as a function of the heat transfer coefficient κ_i between the gas and the skeleton [of the porous plates]. Results are shown in Table 1.

Figure 1 shows graphs of δT as a function of κ_i. The calculations were conducted by varying DT, the increment in the gas temperature due to the chemical reaction.

TABLE 2. Variation of the Gas Mass Flow G and the Heat Transfer Coefficient κ_i for the Following Parameter Values: $DT/T_0 = 2$, $\varepsilon_i = 0.7$, $\Pi_i = 0.8$, $T_0 = 300$ K, and $L_i = 0.01$ m

G, kg/(m²·sec)	0,003	0,005	0,01	0,03	0,05
			$\varkappa_i = 10^3$ W/(m³·K)		
T_{max}/T_0	3,509	3,409	3,252	3,158	3,094
T_{out}/T_0	1,704	1,029	2,428	2,580	2,619
			$\varkappa_i = 10^5$ W/(m³·K)		
T_{max}/T_0	3,942	4,226	4,470	4,387	4,008
T_{out}/T_0	1,225	1,488	1,729	2,152	2,352

TABLE 3. Variation of the Gas Mass Flow G and DT for the Following Parameter Values: $\kappa_i = 10^4$ W/(m³·K), $\varepsilon_i = 0.7$, $\Pi_i = 0.8$, $T_0 = 300$ K, and $L_i = 0.01$ m

G, kg/(m²·sec)	0,001	0,005	0,01	0,03	0,05	0,1
			$DT/T_0=1$			
T_{max}/T_0	2,380	2,634	2,517	2,207	2,125	2,058
T_{out}/T_0	1,084	1,374	1,534	1,780	1,858	1,929
			$DT/T_0=2$			
T_{max}/T_0	3,596	4,093	4,116	3,595	3,369	3,185
T_{out}/T_0	1,144	1,579	1,879	2,389	2,587	2,775
			$DT/T_0=3$			
T_{max}/T_0	4,750	5,383	5,534	5,042	4,685	4,354
T_{out}/T_0	1,192	1,716	2,097	2,855	3,202	3,555
			$DT/T_0=4$			
T_{max}/T_0	5,873	6,595	6,824	6,725	6,005	5,979
T_{out}/T_0	1,234	1,822	2,257	2,829	3,741	3,890
			$DT/T_0=5$			
T_{max}/T_0	6,975	7,766	8,048	7,785	7,301	6,722
T_{out}/T_0	1,271	1,910	2,386	3,029	4,223	4,985

In Fig. 1a, the ordinate is the quantity δT, normalized to the initial gas temperature T_0. For $DT/T_0 \geq 3$, saturation starts for $\kappa \geq 10^6$ W/(m³·K). The value of κ was not increased further, because $\kappa \geq 10^6$ W/(m³·K) is difficult to realize physically. From the figure it is clear that increasing DT leads to a growth of δT.

Figure 1b shows the dependence of the relative increase $\delta T/DT$ on κ for various energies released in the working volume. The value of $\delta T/DT$, that is, the "efficiency" of introducing energy into the working volume, initially grows as DT is increased, but then starts to fall off. Calculations were conducted for DT = 300, 600 900, 1200, 1500, and 2100 K, while the heat transfer coefficient was varied from 10^3 to 10^6 W/(m³·K). The optimum $\delta T/DT$ was the regime with DT ≈ 1500 K for $T_0 = 300$ K.

Tables 2 and 3 show the maximum and output gas temperatures in the system of porous plates for variations in the gas mass flow. It can be seen that increasing the gas mass flow leads to a growth in δT, which initially grows, but then starts to decrease. There are optimum gas flows for various values of κ_i and DT, at which δT is maximized. As DT is increased, the maximum value of δT increases and shifts towards larger gas mass flows.

Figure 2 shows δT as a function of G for three values of κ_i: 10^3 10^4, and 10^5 W/(m³·K). It can be seen that as κ_i increases, the optimum shifts to larger gas flows G.

The behavior of the temperature curves for the gas and the skeleton does not change in the first plate; the temperature functions are always monotonic. In the second plate,

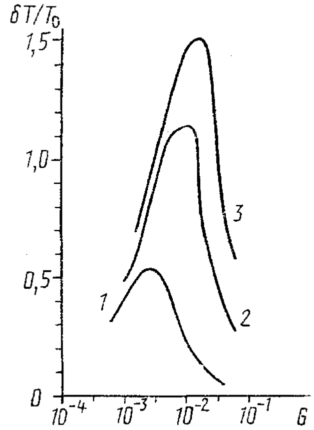

Fig. 2. Gas temperature growth $\delta T/T_0$ as a function of the gas mass flow G [kg/(m²·sec)] for various values of κ_i: 1) $\kappa_i = 10^3$ W/(m³·K); 2) $\kappa_i = 10^4$ W/(m³·K); 3) $\kappa_i = 10^5$ W/(m³·K).

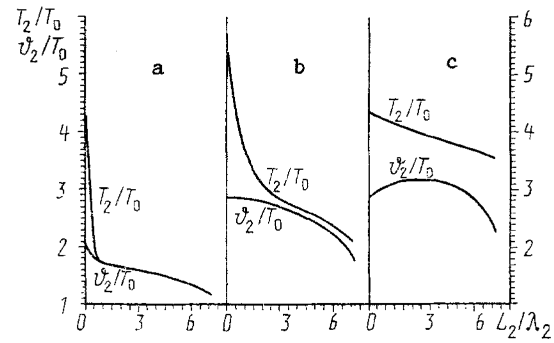

Fig. 3. Profiles of the gas temperature T_2 (K) and the skeleton ϑ_2 (K) for various gas mass flows for DT = 1500 K, and $\kappa = 10^4$ W/(m³·K): a) G = 0.005 kg/(m²·sec), $G < G_{opt}$; b) G = 0.01 kg/(m² × sec), $G = G_{opt}$; c) G = 0.1 kg/(m²·sec), $G > G_{opt}$.

depending on the magnitude of the flow, three cases can be distinguished, in which the shape of the temperature curves differs qualitatively. Figure 3 shows the temperature profiles of the skeleton ϑ_2 and the gas T_2 for three gas flow regimes: less than optimum, optimum, and greater than optimum. From the figure it can be seen that the derivative of the skeleton temperature with respect to x at the plate input is negative for $G < G_{opt}$, is zero for $G = G_{opt}$, and positive for $G > G_{opt}$.

Varying the emissitivity ε_i does not make any noticeable changes in the picture. It is obvious that the effect of the emissivity ε_i of the particles which make up the skeleton of the porous plates is substantial only for the transient regime.

Figure 4 shows the quantity $\delta_1 T = (T_{max} - T_0)/(T_{ad} - T_0)$ as a function of the dimensionless optical thickness ℓ for the following parameter values: $\kappa_i = 5\cdot10^4$ W/(m³·K), G = 0.04 kg/(m²·sec), T_0 = 297 K and DT = 900 K. Curve 1 corresponds to our results; curve 2 is taken from [1]. The fact that our curve does not become stationary and that its maximum temperature exceeds the analogous temperature from [1] is explained by the fact that our proposed model does not consider thermal conductivity through the gas and between layers of the porous plates. It is obvious that considering these processes will lead to a more even heating of the porous plates. From Fig. 4 it can be seen that the rate of growth of curve 1 decreases as the optical thickness is increased above 2.5 photon mean free paths. Increasing the thickness of the first plate above 15 mean free paths does not improve the efficiency. This evidently is completely sufficient to trap the radiation of the particles

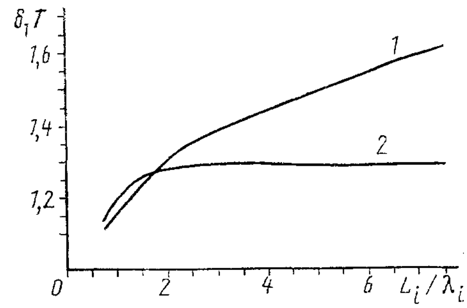

Fig. 4. Gas temperature increment $\delta_1 T/T_0$ as a function of the total thicknesses of the plates. The thicknesses of both plates are identical.

of the second plate in the direction of the first. The thickness of the second plate also cannot grow without bound, because losses to radiation energy in the direction parallel to the plate surface produces an undesired effect.

Finally, we will formulate basic conclusions which result from solving the problem. Numerical modeling showed that increasing the heat transfer coefficient κ_i shifts the optimum towards an increased gas mass flow G. There is also an optimum value for the energy added to the working volume. The optimum gas mass flow was shown. For reasonable physical parameters, for example for $\kappa_i = 10^4$ W/(m^3·K), the optimum flow is 0.01 kg/(m^2·sec).

NOTATION

L_i, thicknesses of the porous layers (i = 1, 2); J_{0i}, radiation flux falling on the plate from the left; J_{1i}, radiation flux falling on the plate from the right; Π_i, porosity of the porous layers; ε_i, emissivity of the particles in the porous layers; C_p, heat capacity of the gas; ρ, density of the gas; $G = \rho u$, gas mass flow; κ_i (volumetric), heat transfer coefficient between the skeleton of the porous plates and the gas; T_0, initial temperature of the gas; T_i, temperature of the gas in the porous plates; ϑ_i, the temperature of the particles in the porous plates; DT, gas temperature increment due to the chemical reaction; δT, gas temperature increment due to heat transfer between the skeleton of the plates and the gas; σ, Stefan–Boltzmann constant; λ_i, mean free path of photons in the porous plates; S_i, radiating surface of a unit volume of the porous plates; and F_i, radiation density leaving a unit volume of the porous plates.

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